EPICYCLIC GEARING AND THE ANTIKYTHERA MECHANISM

PART I

by M. T. Wright

THE oldest known use of toothed gearing, the Antikythera Mechanism, is preserved in the National Archaeological Museum in Athens.[1] It became widely known through the work of the late Professor Derek Price, whose monograph *Gears from the Greeks* remains the best general introduction to the subject.[2] Price suggested that the Mechanism, dating from the first century BC, demonstrated the existence of a highly-developed Hellenistic tradition of geared instruments which, transmitted to and preserved in Arabic culture, thence influenced the development of the Western European tradition of clockwork.

This argument remains unchallenged. I have myself contributed to work that provides good evidence for the earlier transmission, from Hellenistic to Arabic culture.[3] On the other hand, a careful reading of his account showed that Price's understanding of the Antikythera Mechanism itself was flawed. This led the late Professor Allan Bromley and myself to make a new, more detailed, survey of the original fragments.[4,5] I am now developing a new reconstruction on the basis of these observations, which differs in many important respects from others which are all based on Price's observations.

I have published a partial reconstruction, demonstrating that one face of the instrument, which Price called the 'front', may be restored as a planetarium.[6] The motions in longitude of the Sun, the Moon and the five planets known in antiquity are shown by seven hands, all controlled by epicyclic mechanism, moving over a Zodiac circle. An eighth hand shows the date on a concentric calendar ring. This scheme accounts better and more fully for some prominent features found on the 'front' face of the original fragment **A** (Fig. 1) than does any other reconstruction known to me, and it is consonant with the reports of planetaria that are found in literature contemporary with the Antikythera Mechanism.[7]

Anticipating doubt that my reconstruction might be thought practicable, I made a working model to illustrate it.[8] Figure 2 shows the external appearance of the model in its present

1. Inventory Number X.15087.
2. D. J. de S. Price: 'Gears from the Greeks', *Transactions of the American Philosophical Society*, Vol. 64 No. 7 (1974). Reprinted as an independent monograph, (New York: Science History Publications, 1975). Also published in Greek by Technical Museum of Thessaloniki (1995).
3. J. V. Field & M. T. Wright, 'Gears from the Byzantines: a portable sundial with calendrical gearing', *Annals of Science* 45 (1985), 87-138; also reprinted in J. V. Field, D. R. Hill & M. T. Wright, *Byzantine and Arabic Mathematical Gearing*, (London: Science Museum, 1985).
4. Allan George Bromley, formerly Associate Professor in the Basser Department of Computer Science, University of Sydney, died on 16th August 2002, aged fifty-five.
5. A. G. Bromley, 'Observations of the Antikythera Mechanism', *Antiquarian Horology*, Vol. 18 No. 6 (Summer 1990), 641-652. M. T. Wright, A. G. Bromley & H. Magou, 'Simple X-ray Tomography and the Antikythera Mechanism', PACT 45, (1995). M. T. Wright & A. G. Bromley, 'Current Work on the Antikythera Mechanism', *proc. conf. Ancient Greek Technology* (Thessaloniki: Technical Museum of Thessaloniki, 1997).
6. The underlying theoretical and historical considerations are covered in two papers: M. T. Wright & A. G. Bromley, 'Towards a New Reconstruction of the Antikythera Mechanism', *proc. conference Extraordinary Machines and Structures in Antiquity,* Ancient Olympia (August 2001), (Athens, Kaktos, forthcoming); and M. T. Wright, 'In the steps of the Master Mechanic', *proc. conference Ancient Greece and the Modern World,* Ancient Olympia, (July 2002), forthcoming.
7. See Price (1974), note 2.
8. The model, in an early form, is described and illustrated in: M. T. Wright, 'A Planetarium Display for the Antikythera Mechanism', *Horological Journal*, Vol. 144 No. 5 (May 2002), 169-173, and Vol. 144 No. 6 (June 2002), 193. In a slightly more developed form, it has now stood the test of being displayed to the Athenian public, who were free to work it, in the exhibition *Ancient Technology at Technopolis*, September-October 2002.

Fig. 1. Antikythera Mechanism, fragment **A**, front. Approximately 35% actual size.

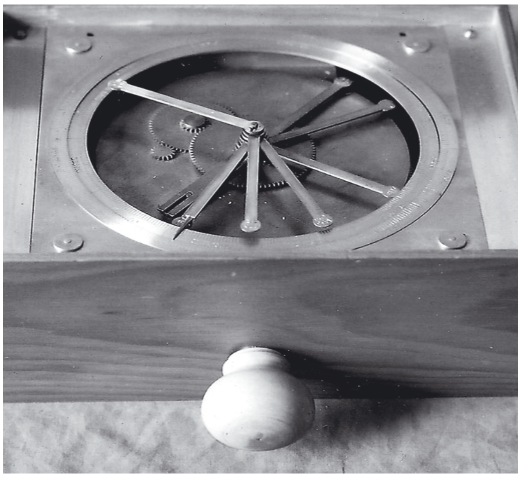

Fig. 2. Antikythera Mechanism, reconstruction of the front dial by M. T. Wright.

state and Fig. 3 shows it partially disassembled so that its relationship to the original, seen in Fig. 1, is clear. In making this model I took care to imitate details seen in the original fragments and to make my conjectural restorations in a similar style. For the most part, I used only simple hand tools. The only important respect in which I used equipment not available in antiquity was in cutting the wheels, for which I saved time by using 'modern' (1887) dividing apparatus. I have, however, previously demonstrated that there is no difficulty in

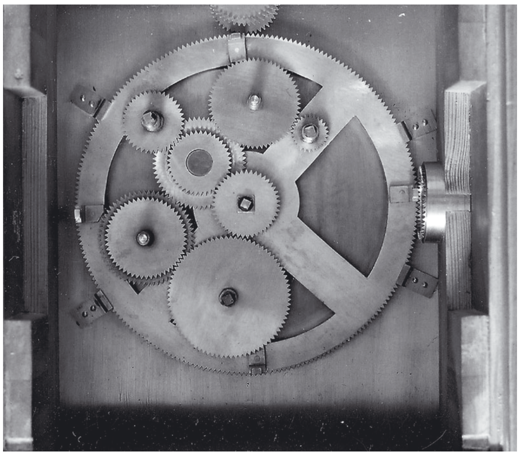

Fig. 3. Model as Fig. 2, partially disassembled to show its correspondence to the original, Fig. 1.

dividing the circle into any desired number of parts and cutting teeth, to a sufficient degree of precision, using only very simple tools.[9]

In its present state my model contains forty-one wheels beyond those for which direct evidence is found in the original fragments, all serving the planetarium dial. Most of them are found within the five separate epicyclic assemblies that give motion to the Sun, Moon and planet hands, and the others drive the three assemblies for the superior planets. This represents, roughly speaking, a doubling of the number of wheels in the whole instrument. The mechanism is not, however, made much more intricate than was previously supposed; rather, it is made more extensive. In particular, the epicyclic assemblies that I introduce are comparable with the one of which about half survives on the rear face of the original fragment **A** (Fig. 4). According to Price, this assembly functioned as a differential gear.

The identification of this sophisticated ensemble within the remains of the earliest geared mechanism known has naturally called forth much speculation and comment. While it provides me with a precedent for the epicyclic clusters which, in my reconstruction, furnish the motion of the pointers for the Sun, Moon and planets according to epicyclic models, I reject a recent suggestion that it might itself have served as part of such a model.[10] Firstly, I cannot see

9. M. T. Wright, 'Rational and Irrational Reconstruction: the London Byzantine Sundial-Calendar and the Early History of Geared Mechanisms', *History of Technology*, Vol. 12 (1990), 65-102.

10. T. Freeth, 'The Antikythera Mechanism', *Mediterranean Archaeology & Archaeometry*, Vol. 2 Nos 1 & 2 (June & December 2002), 21-35 & 45-58.

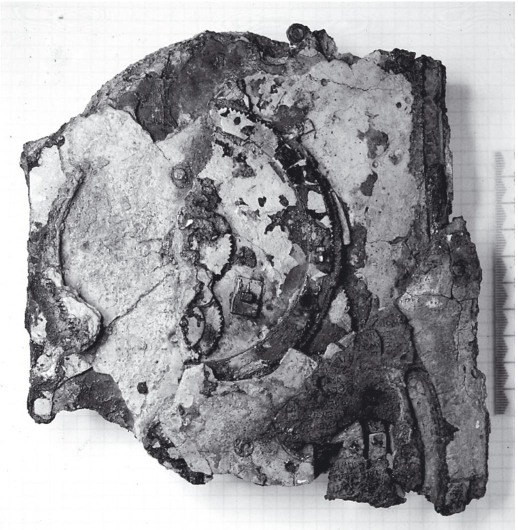

Fig. 4. Antikythera Mechanism, fragment **A**, back. Approximately 35% actual size.

how to interpret the visible detail in this way. Secondly, it is clear, both from direct observation and from radiographs, that the epicyclic assembly has not been significantly displaced from its intended position with respect to the frame plate (Price's 'base plate') on which, along with most of the other mobiles, it was planted; it lies between this plate and the back dial plate, and it is not concentric with any of the dial systems of the instrument. Thirdly, the function of the surviving gearing of the front dial, and its connection to the epicyclic gear (which will be discussed below), both seem sufficiently clear to confirm that this epicyclic platform cannot have turned at a rate appropriate to the realisation of any epicyclic astronomical model.

In fact, my reconstruction of the front dial depends crucially on the observation that the

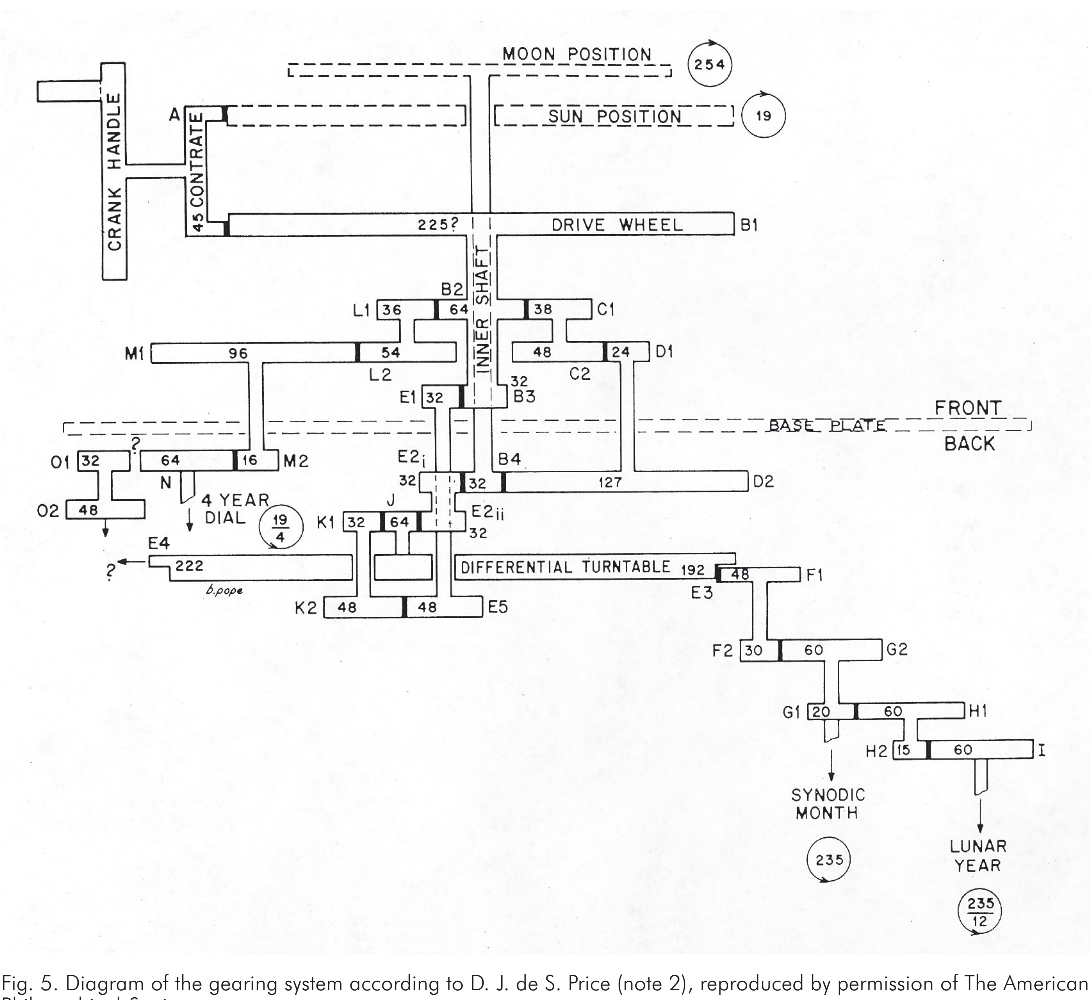

Fig. 5. Diagram of the gearing system according to D. J. de S. Price (note 2), reproduced by permission of The American Philosophical Society.

gearing, within the mass of fragment **A**, that connects the front dial to this epicyclic assembly, is not as Price suggested. I have drawn attention to this fact on several occasions before now.[11] I will however demonstrate the correct arrangement in greater detail below because, while this modification opens the way to my reconstruction of the front dial, it also obliges us to reconsider the arrangement and function of other parts of the Mechanism, and especially of the supposed differential gear. It is with this part of the mechanism that the present paper will be largely concerned. Although the thrust of this paper is to show that there are several respects in which Price's reconstruction cannot be correct, the widespread acceptance of this reconstruction makes it a convenient starting point.

PRICE'S RECONSTRUCTION

Price's sectional diagram of the gearing system of his reconstruction is reproduced as Fig. 5.

The instrument is driven by turning wheel **A**. At the top is the 'front' dial, centred on axis **B**, on which indicators show the mean positions of the Sun and the Moon in the Zodiac.[12] The former also shows the day of the year. The two indicators are connected by gearing, through axes **C** and **D**, which embodies the period relation (19 tropical years) = (254 tropical months).[13] The motions of these two mobiles are transmitted to axis **E** where they are combined in an epicyclic differential gear assembly, the output of which leads, through wheels on axis **F**, to drive a pointer on axis **G**, the centre of the 'lower back' dial, and thence through axis **H** to another pointer on axis **I**, the centre of a subsidiary dial.

Another train leads from axis **B** through axes **L** and **M**, towards the 'upper back' dial at axis **N** and subsidiary dial at axis **O**. Beyond axis **M**, however, the arrangement is uncertain, and I leave the problems of this part of the instrument aside for now. Here I am concerned particularly with the train planted on axes **E**, **F**, and **G**.

Price's differential gear is a prominent and remarkable feature. Wheel **E1** is turned by **B3** at the rate 1/(tropical year), and **E2i** is turned by **B4** at 1/(tropical month), in the opposite sense. The platform, wheels **E3** and **E4**, turns at half the difference of the two inputs, which is 1/(2 synodic months). This rate of rotation is doubled and transferred to the lower back dial by the fixed-axis train of spur gears, **E3-F1-F2-G2**, so that the pointer on axis **G**, working on the lower back dial, turns once in a synodic month.

I will show that, in developing this scheme, Price allowed himself very considerable latitude in interpreting the evidence. Moreover, his interpretation represents a remarkably complicated way of obtaining a display of the synodic month. The same well-known period relation embodied in the train from **B2**, through axes **C** and **D** to **B4**, which, as Price showed, gave the designer 19 years = 254 tropical months, also offered him 19 years = 254-19 = 235 synodic months (the Metonic ratio). In other words, the designer could easily have obtained an output displaying the synodic month by working with plain, fixed-axis gearing, using wheels of moderate size (because 235 = 5 × 47), from axis **D**. Bromley made this point forcibly by designing an instrument that offered the same outputs using just 12 wheels.

One may take such an argument much further: a display of the synodic month could have been added to the front dial, using no further wheelwork at all, just as it is done in the dial work of many astronomical clocks. Direct use can be made of the relative motion of the two indicators showing the celestial longitudes of the Sun and the Moon; a fiducial mark fixed to one, moving over a concentric scale attached to the other, shows the age of the Moon. The scale is usually divided into 29½ days, and,

11. I first made this point formally in: M. T. Wright & A. G. Bromley, 'Current Work on the Antikythera Mechanism', *proc. conference Ancient Greek Technology* (Thessaloniki, 1997).

12. It is on this dial that I have developed my planetarium display.

13. Price refers to the motion of the Moon against the stars, and to the siderial [*sic*] month. Since, however, the relative motion of the equinox and the stars had been noticed (if not understood), and the pointers work over a scale of celestial longitude, not a star-map, it seems more proper to take the rotation of the Moon pointer as representing the tropical, not the sidereal, month. Correspondingly, the rotation of the Sun pointer is taken to represent the tropical year, which was, in any case, a fundamental quantity for Hipparchus. The numerical consequences of the change are, of course, slight, and may be negligible.

although this is a crude approximation, no error accumulates from month to month.[14] A representation of the phases of the Moon is often achieved by fitting a concentric disc, having an eccentric circular aperture, to the outer of the two indicators, through which, usually, one sees a portion of a contrasting eccentric zone on a further disc attached to the other indicator. Occasionally, the opening contains a rotating Moon ball.[15] It is possible that the 'drumlike component' reported by Price (note 2, 20), which I measure as 62-63 mm. in diameter, may be the relic of some such display feature, but I have not so far developed a wholly satisfactory reconstruction of it.[16] Such an arrangement could be applied equally well either to Price's front dial display or to mine. Applied to my reconstruction which models the motions of the Sun and Moon according to the theories of Hipparchus, this display would give a superior performance to any that is based merely on the mean motions of the Sun and Moon.

Of course, the fact that Price's reconstruction achieves a display that could have been realised by a simpler arrangement is by no means a sure indication that it is wrong. It is, in fact, not uncommon for an early essay in a developing technology, which the Antikythera Mechanism might arguably be supposed to represent, to be more complex than necessary. The contrast between the evident accomplishment of the execution of the original instrument and over-complicated way in which Price's reconstruction achieves the supposed dial displays does, however, give one pause for thought.

Besides this, there is a practical problem with Price's reconstruction. The whole system is driven from the slow-moving end of the train.[17] The overall effect is of a branched step-up train yielding ratios of roughly 12.4 : 1 and 13.4 : 1. Ordinarily this would pose no difficulty to the horologist, but it should be remembered that the gears in this instrument are formed with roughly triangular teeth and that other mechanical details are naïve. In particular, the loads developed within the differential assembly lead to high friction, and a recent commentator has gone so far as to suggest that the arrangement is unworkable.[18] This is an exaggeration, but it is commonly admitted by those who have built models of it, even the majority in which the wheelwork has been realized using modern details, that it is not easy to make this arrangement work. The problem is exacerbated when the attempt is made to copy the details of the original more closely.

Bromley's modification of Price's scheme avoided this problem by driving the train from gear **E4**, for which Price had found no purpose.[19] **E4** is the rim of the relatively fast-moving epicyclic platform itself, and, worked in this way, the instrument runs very sweetly. Bromley drove **E4** through a reduction train which began with a knob or winch turned (approximately) once per day, an astronomical period that, he suggested, might appropriately

14. The earliest example of this device is found as an addition to a planispheric astrolabe dated to *c.* 1300, in the collection of The Science Museum, London (Inventory Number 1880-32). The arrangement is now incomplete and garbled. A reconstruction is offered in J. D. North, 'Opus quorundam rotarum mirabilium', *Physis*, 8 (1966), 337-371.

15. Examples are described and illustrated in H. C. King & J. R. Millburn, *Geared to the Stars* (Toronto, Buffalo & London, 1978). The earliest example of any portrayal of the phases of the Moon effected by a design seen through an aperture is found in the London Byzantine Sundial-Calendar (Science Museum, London, Inventory Number 1983-1393), which came to light after this book was written. See note 3.

16. Price himself made this suggestion tentatively, before venturing as an alternative that it might have been a (folding) 'crank handle' [*sic*] for working the instrument. The latter interpretation, which is certainly wrong, has unfortunately been widely accepted. There is actually much detail within this structure to be explained. See Wright, Bromley & Magou (1995) and Wright & Bromley (1997), note 5.

17. This point was well expressed by Zeeman: E. C. Zeeman, 'Gears from the Greeks', *Proceedings of the Royal Institution of Great Britain*, Vol. 58 (1986), 137-156.

18. G. White, 'Antikythera Gearing - a different solution', *Horological Journal*, Vol. 144 (October 2002), 358-363. White's main point in this article is an attempt to rationalise the features of Price's reconstruction by suggesting alterations to it. Unfortunately, because he relies on elements of this reconstruction that I show cannot be accepted, his ingenuity is wasted.

19. A. G. Bromley, 'The Antikythera Mechanism', *Horological Journal*, Vol. 132 No. 6 (June 1990), 412-415, and Vol. 133 No. 1 (July 1990), 28-31.

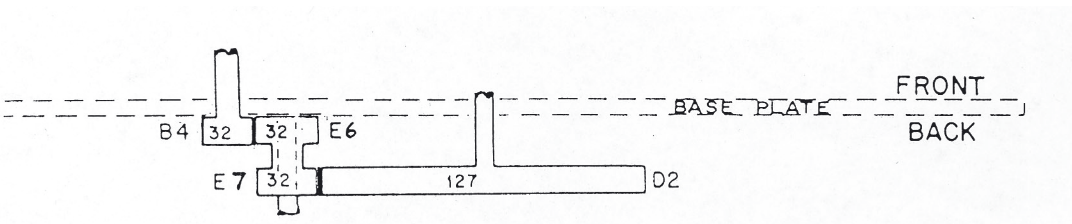

Fig. 6. A first modification of Price's gearing system: cf. Fig. 5.

be represented in the instrument.[20] This variant has gained some currency through being published and exhibited, but it must be understood that Bromley developed it before he had inspected the original fragments, and that it was wholly conjectural. On obtaining access to the original he tried, and failed, to find a way in which such a driving train might have been fitted. Furthermore, since the day is clearly indicated on the front dial, Bromley's argument for his 'one turn per day' input was not as strong as he claimed. Finally, I have stated, and will show in greater detail below, that the large wheel equal in size to **B1** (shown dotted and marked 'Sun Position' in Fig. 5), which Price introduced to engage the upper limb of contrate wheel **A** as a reversing arrangement, is not needed. This leaves no other function for wheel **A** than as the mobile by which the instrument is driven.

Through what follows it will be found progressively harder to continue to accept Price's reconstruction, or, by the same token, any other reconstruction based on it. That is to say, all reconstructions of the Antikythera Mechanism known to me, other than the one that I am developing, must be abandoned. In particular, I show in this first part of my paper that the differential gear cannot have functioned as Price suggested, and so it is highly doubtful that the synodic month was displayed on the lower back dial.

MODIFICATIONS TO PRICE'S SCHEME

I draw attention to wheels **E2i**, **B4** and **D2** in Fig. 5, and invite comparison with the true arrangement shown in Fig. 6. In reality, wheel **D2** does not engage **B4** but engages what I call wheel **E7**. (The rationale for the nomenclature is explained below.) **E7** lies adjacent to another wheel, marked in Fig. 6 as **E6**, which in turn engages wheel **B4**. These wheels are seen in their true spatial relationship in my model, Fig. 7. **E6**, **E7** and **B4** each have 32 teeth. **B4** and **E6** are larger and coarser in pitch than **E7** (and, of course, **D2**), so that **E6** is overlapped by the edge of wheel **D2**. Readers who may hope to check details of the arrangement in plan, using the gearing diagrams given by Price (note 2, his figures 29 - 31), should be aware that in some respects these drawings are seriously at variance with reality.

The effect on the train connecting wheels **B1** and **B4** is as though an idle wheel had been introduced at some point, so that **B1** and **B4** now rotate in the same sense. If the rotation of the central arbor bearing wheel **B4** still, as before, represents the motion of the Mean Moon, the rotation of **B1** may now represent the motion of the Mean Sun, without Price's clumsy reversing arrangement. The removal of the duplicate of **B1**, engaging the upper limb of wheel **A**, opened the way to my new reconstruction of the front dial.[21]

Were all the other connections to remain unchanged, the modification illustrated in Fig. 6 would also reverse the sense of rotation of the wheels on the hollow arbor at axis **E** (**E2i** and **E2ii** in Fig. 5), with respect to the central arbor with wheels **E1** and **E5**. Price's scheme for the back would then be unworkable, because the

20. Some commentators state that Bromley's day input is 'exact'. However, as Bromley himself pointed out, due to the incommensurability of the day and the year, the velocity ratio of his driving train is an approximation. It follows that the inexactitude of this approximation must be reflected either in the value of the day or in the values of all the other periods displayed by the instrument. Whether or not it is proper to regard Bromley's one-turn-per-day as exact therefore depends on the use to which we imagine the instrument being put.

21. In this I continue to accept, at least provisionally, Price's counts for the wheels **B2**, **C1**, **C2**, **D1**, **D2** and **B4**. This leads me to agree with his conclusion as to the ratio of the periods of rotation of **B2** and **B4**, although I prefer to express it as (tropical year) : (tropical month). (See note 12.)

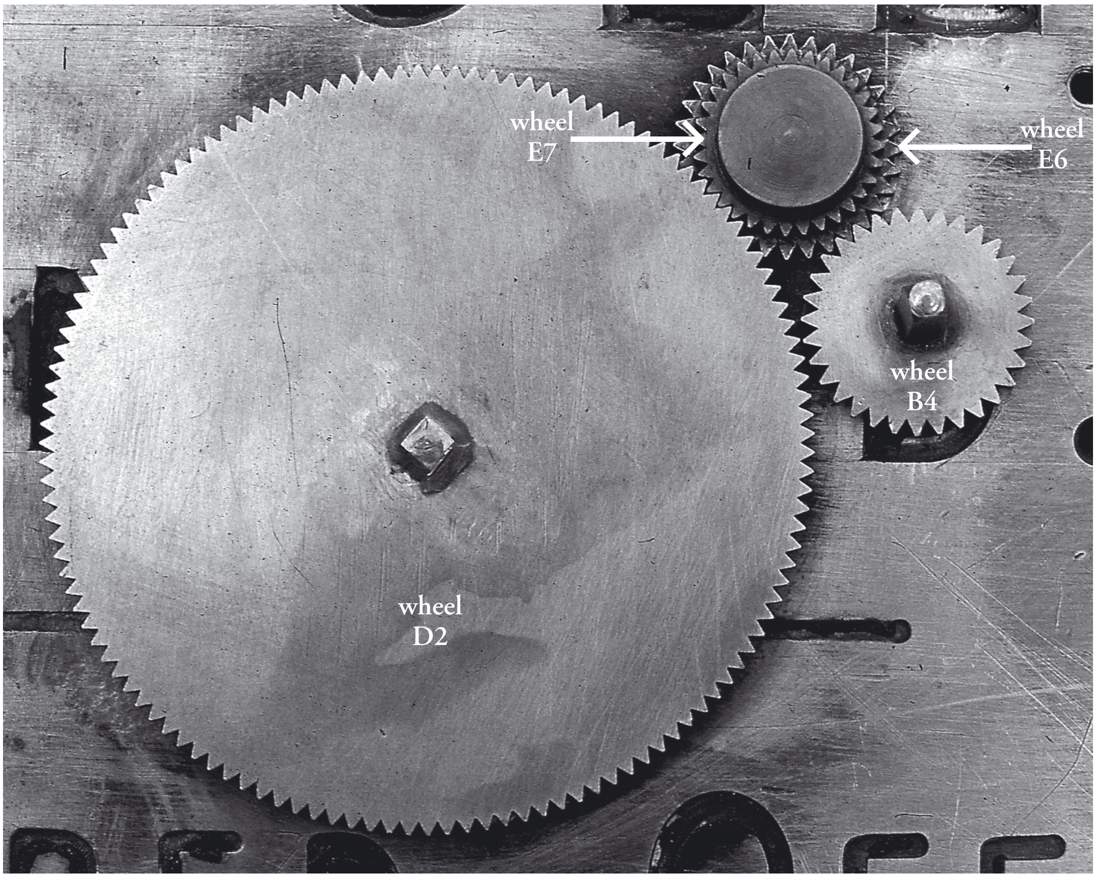

Fig. 7. Model as Fig. 2, in an early experimental state. Underside, showing gears **B4** (right), **E6** and **E7** (upper right), and **D2** (left) in their correct spatial relationship. In its present state the model does not include the epicyclic platform which should be centred on axis **E**. Certain features seen in this view, which are not found in the original, are present because this model was built of scrap. Aproximately 160% actual size.

two inputs to the differential assembly would turn in the same sense; the rate of rotation of the output would accordingly be half the sum, not half the difference, of the rates of rotation of the two inputs, a quantity of no interest in this context.

The radiographic survey that leads me to insist on the rearrangement illustrated in Figs 6 and 7 yields evidence, however, that Price's scheme must be adjusted yet further. In place of wheel **E1** there is only a roughly circular 'button' with no teeth, and wheel **B3** cannot be found at all. There is therefore no extant second input to Price's differential gear. It is possible to contrive such an input, turning in the sense now required, as a conjectural restoration. In doing so, however, we would be obliged to postulate the loss of several wheels from the original, some of them in places from which they could not simply have dropped out. While such losses are conceivable, I consider them unlikely. Moreover, the change to the overall velocity ratio of the fixed-axis train following the epicyclic assembly, which, as I show below, is forced on us by a rereading of the evidence, dictates that no such 'quick fix', retaining the function of Price's differential gear unaltered, can be satisfactory.

Instead, I suggest that the missing second input to the 'differential gear' may never have existed, that the central arbor at axis **E** was fixed to the base plate, and that wheel **E5**, mounted on it, was stationary. The epicyclic assembly would then have functioned not as a differential gear but as a gear train with one input and one output.

This more extensive rearrangement is illustrated in a further partial revision of Price's sectional diagram, Fig. 8. I begin by pointing out that Price was mistaken, in showing wheels on the underside of the epicyclic platform (**E2ii**,

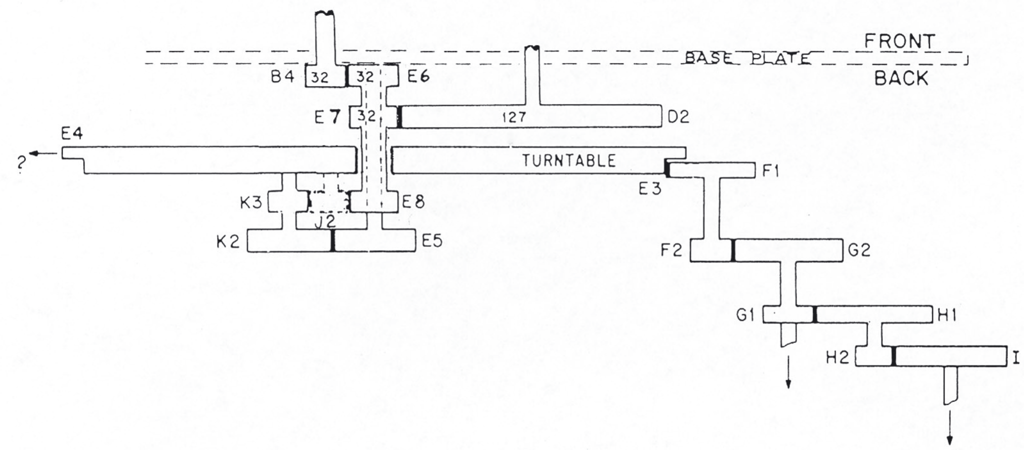

Fig. 8. A further modification of Price's gearing system: cf. Figs 5 & 6.

K1 and **J** in Fig. 5) that are in reality found above. This particular error makes no difference in principle to the working of the epicyclic assembly, but in Fig. 8 the wheels, renamed **E8**, **K3** and **J2**, are shown in their proper places. The other respects in which Fig. 8 differs from Fig. 5 are more important.

Price's convenient system of nomenclature (letter for axis, number for wheel) is retained, but it is modified. The tidiest result would have been achieved by reallocating the numbers (such as **E1**, **E2**, ... in the case already discussed), but this might have led to confusion in comparing my revised scheme with that of Price. Instead, where Price showed a wheel, the presence or function of which I cannot verify, I do not use that letter/number code. Thus, I have no **B3**, **E1** or **E2**. Wheels that are moved are treated in the same way as new introductions, with a new letter/number code. Thus, Price's wheels **E2ii**, **J** and **K1** below the turntable are replaced by **E8**, **J2** and **K3** above it. Price's **E2i** becomes **E6**.

Note that in Fig. 8 I indicate that axis **J**, carrying an idle wheel, is a conjectural restoration. Price stated as much in his text, but did not indicate it in his diagram. The possible importance of this point will be seen later.

The tooth-counts of the wheels, and the question as to whether axis **J** existed at all, will be discussed in the second part of this paper. Here I limit myself to pointing out that Price was obliged to invent wheel **K1** (for which there is no evidence) equal to **E2ii** (which again does not exist, but which was plausible on the basis of the evidence available to him), and to overrule the observations of the radiographer Karakalos (whose plates he used) that **E5** and **K2** were 'double wheels', probably having unequal numbers of teeth, because he required equal wheels on axes **E** and **K** at each level within his differential assembly. On the other hand, in the arrangement now outlined, according to which we see this assembly as an epicyclic gear with a fixed central arbor, it would make no sense to have equal wheels at both levels: with an idle wheel at **J**, the assembly would then yield the trivial overall gear ratio of 2 : 1, making its presence pointless; without the idle wheel, and with **E8** and **K3** engaging one another directly, the input would be locked.

THE EPICYCLIC TRAIN: GENERAL CONSIDERATIONS

The epicyclic arrangement shown schematically in Fig. 8 is highly reminiscent of the trains used in the late eighteenth century by Fr. David a Sancto Cajetano, who published accounts showing how epicyclic trains might be designed to yield desired velocity ratios that could be obtained only with difficulty, if at all, using conventional fixed-axis gearing alone.[22] No earlier analysis of epicyclic gearing is known.[23] However, several earlier examples of epicyclic

22. Fr. David a S. Cajetano: *Neues Rädergebäude* (Vienna, 1791); *Praktische Anleitung für Künstler ...* (Vienna & Leipzig, 1793); and *Neues Rädergebäude mit Verbesserungen und Zusätzen* (Vienna & Leipzig, 1793 & 1794).

23. G. White, 'The Precise Epicyclic Gears of Fr. D a San [*sic*] Cajetano', *Horological Journal*, Vol. 135: No. 2 (August 1991), 65-68; No. 3 (September 1991), 86-89; No. 4 (October 1991), 137-140.

TABLE 1

wheel	**E3**	**F1**	**F2**	**G2**	**G1**	**H2**	**H1**	**I**
Karakalos	192	54	30	54/55	20	16	60-62	60
Price	192	48	30	60	20	15	60	60
Wright	192	54	30	54	20	15/16	60	60?

gearing survive, besides the one considered here, and I suppose that the designer of each must have had a sufficient understanding of, and workable method for analysing the performance of, what he created. Just what analysis might have been achieved when the Antikythera Mechanism was designed is a question for the historian of mathematics.

Unfortunately, what remains of the epicyclic gear within the Antikythera Mechanism is so poorly preserved that it is difficult to reconstruct it, and to explain its function on the basis of its internal arrangement, with any degree of certainty. If this could be done, it might shed light on the probable method of analysis used in its design. Conversely, were the method of analysis known it might be used as a tool in reconstructing the mechanism. As it is, we can do little more than weigh probabilities, so that the reconstruction will remain uncertain and the method by which it was designed a matter of speculation.

Detailed observations of the epicyclic gear in the Antikythera Mechanism are reserved for the second part of this paper, in which I will consider the range of possibilities for its reconstruction and offer a tentative conclusion. Here I will show that some further progress may be made by considering the context in which it is found.

THE FIXED-AXIS TRAIN TO THE LOWER BACK DIAL

Leaving aside the uncertainties of the epicyclic gear itself, I turn my attention to the fixed-axis train between it and the lower back dial. This comprises **E3** (the gear-ring on the epicyclic platform) and the wheels on axes **F**, **G**, **H** and **I**. As elsewhere, Price had at his disposal the tooth-counts of Karakalos. He reported them but he was particularly embarrassed by them because they did not yield the simple 2 : 1 ratio between **E3** and axis **G** that he required, as described above. He therefore changed the counts of wheels **F1** and **G2** by about six teeth in each case, decreasing the first and increasing the second.

A few wheels in the surviving fragments are so poorly preserved that there is great uncertainty in counting their numbers of teeth. The wheels in this train do not, however, offer so much latitude as to justify the licence that Price took. I have checked the counts myself using my own radiographs, and I give my counts, together with those of Karakalos and Price, in Table 1. I list the wheels in the order in which they appear in the train.

Of these, the count for **F2** is quite secure because its complete periphery can be seen in radiographs. The uncertainties in the numbers of teeth of the others are also strictly limited.

For example, as mentioned above, only about half of **E3** is present. Moreover, several small pieces of the rim of this half are broken out. On the other hand, a small piece remains roughly opposite the centre of the preserved half, which provides the positions of a few teeth and, more importantly, a secure means of finding the centre. The need to have a whole number of teeth in each interval provides a powerful constraint, and the results of repeated trials using various procedures are consistent.

There are sources of potential error, including possible lack of uniformity in division, uncertainty in finding the centre, and out-of-roundness of the image of the wheel, whether through having never been circular, through damage, or through lying oblique to the plane of the radiograph. The effect of such factors may be large when only a small part of the periphery of the wheel, all to one side of the axis, can be traced. None of these wheels, however, seems to present extreme difficulty.

Therefore my tabulated counts cannot be far wrong. One point that may be made firmly is that Price's alteration of the counts of **F1** and **G2**, made simply to suit his scheme, is not only unsupported by, but is actually in conflict with, the evidence. It is clear that the overall velocity ratio for this part of the train as given by Price, exactly 2, must be abandoned. With my numbers, it becomes:

$192/54 \times 30/54 = 160/81 = 1.\dot{9}7530864\dot{1}$

Following the uncertainty admitted by Karakalos, the velocity ratio might instead be:

$192/54 \times 30/55 = 64/33 = 1.\dot{9}\dot{3}$

One should consider the possibility, here as elsewhere in the mechanism, that one or more wheels could have been replaced by others having incorrect numbers of teeth; such interventions by the unintelligent repairer or would-be improver are not unknown. In this case, however, in order to 'restore' the velocity ratio that Price requires we should have to accept that two wrong-numbered replacements had been substituted for original wheels, and that in both cases the change in the tooth-counts was such that some adjustment of the wheels that they engage would probably have been called for, in order to obtain a satisfactory action. Otherwise, modification of the velocity ratio is plausible only if it is based on very small variations of the tooth-counts given above.

The velocity ratio of the fixed-axis train may not, in itself, offer any guidance as to the overall velocity ratio of the suggested arrangement, comprising an epicyclic train followed by the fixed-axis train. We may note, however, that this interim conclusion concerning the fixed-axis train reduces still further the probability that the epicyclic assembly could have functioned as a differential gear, as envisaged by Price. Were it to have done so, the output at the lower back dial would have had a period, corresponding to the reduced velocity ratio, of rather more than one synodic month, a quantity that seems to be of little significance.[24]

FUNCTION OF THE EPICYCLIC GEAR

I consider that the reconstruction of the epicyclic assembly as a differential gear must be abandoned. It seems less problematic to explain its presence as an attempt to derive, in combination with the fixed-axis train that follows it, a desired velocity ratio for which the designer could find no convenient approximation using fixed-axis gearing alone.

The input to this assembly would have had a period of one tropical month, or, more precisely (but still provisionally), 19/254 of the tropical year. Applying again, to this arrangement, the argument that the corresponding period for the synodic month (19/235 of the tropical year) could readily have been obtained by simple means from the gearing of the front dial, it now appears yet more improbable that a display of the synodic month should have been intended at the lower back dial.

The lower back dial itself does not offer us much obvious help in trying to work out what may have been displayed on it, but it is indeed probable that its complicated structure indicates some more sophisticated use than a mere display of the events of the synodic month.[25] The presence of the subsidiary dial may, of course, offer a clue, but it will be seen by inspection of Table 1 that the ratio between the periods of the indicators on the main and subsidiary dials is not certainly 1 : 12 as Price suggested.

I will return to a fuller discussion of the rear dials themselves later. So far as the lower back dial is concerned, this will be informed by the range of possibilities for reconstruction of the epicyclic gear, which I will discuss in the second part of this paper.

24. I am aware that by adopting the second velocity ratio given above ($1.\dot{9}\dot{3}$) for the fixed-axis train, and retaining the function of the epicyclic assembly as a differential gear, one would obtain a period close to one-twelfth of a calendar year for axis G, so that, with the 1 : 12 ratio between axes **G** and **I** suggested by Price, the indicator on the subsidiary dial would rotate with a period close to one year. This seems, however, a far-fetched justification for a scheme that is hard to support on other grounds.

25. I have pointed out that both back dials are constructed of sets of concentric rings, held together by radial bridge-pieces in such a way as to leave concentric tracks in which marker beads (like those of an abacus) or other movable pieces might have slid. See Wright & Bromley 2001, note 5.